Introduction

Welcome to the first meeting of Berkeley's Undergraduate Philosophy of Mathematics Reading Group (unless some other undergraduate has organized a reading group around the topic at Berkeley, in which case, just append to the name the successor of the number of already existing such groups). We're not officially a club, at least not right now, so we don't get any support or encouragement from anyone (exceptions to this are Prof. Mancosu – who is teaching a course on Philosophy of Mathematics this semester by the way, Janet Groome – who is helping us get a permanent room for our meetings). So we gather here (or somewhere else) every week to talk about philosophy of math and logic or related areas. I have followed Prof. Mancosu's advice and decided to organize the meetings around a collection of classic writings on philosophy of mathematics compiled by Benacerraf & Putnam titled *Philosophy of Mathematics: Selected Readings* (2^{nd} Edition). The collection is divided into four parts:

1. Foundations (What is the nature of mathematics?)
2. Ontology (What is the nature of mathematical objects?)
3. Truth (What makes mathematical truths so special?)
4. Sets (What are sets? What is the continuum hypothesis?)

We're already in the fourth week of the semester, we have only about ten meetings ahead of us, but the collection has about twenty-eight papers, so we have to pick and choose. Although all four areas are of immense importance, to go into any of them in any depth we have to ignore some, so what we'll do is focus on the first three parts (Foundations, Ontology, Truth), ignoring the philosophical issues related to sets and the continuum hypothesis. That gives us about three papers on each topic. This week we'll be reading a short piece by Haskell Curry on his formalist conception of mathematics. The next two meetings will be dedicated to intuitionism and logicism respectively. The choice to start with Curry's formalist position is arbitrary and probably pedagogically unsound.

§1 Meeting 1 – Curry on the Nature of Mathematics

This week we'll be reading Haskell Curry's "Remarks on the Definition and Nature of Mathematics." It's a very short piece, but has some interesting things to say about the nature of mathematics as Curry sees it. In this handout I want to highlight some of what I take to be his main points and raise some possible questions for discussion.

Introduction. In the introductory paragraph Curry tells us that the paper is a discussion, from the point of view of a mathematician about some issues in the philosophy of mathematics. This is supposed to excuse his later unfortunate use of the word 'realism', for example, to describe the view according to which mathematical truths are empirical generalizations and so on; whereas, we now would use the word 'realism' to characterize views according to which mathematical objects are real or do exist in some relevant domain. He ends the introduction with a nice summary of his overall position, which he calls *empirical formalism*:

1. Mathematics is an *objective science* with a minimum of *philosophical assumptions*
2. Mathematics is a *body of propositions* dealing with a certain *subject matter*
3. Mathematical propositions are *true* insofar as they *correspond* with *the facts*

According to (1), mathematics is an objective science (like the natural sciences) and therefore it should be based on empirical evidence or facts as opposed to philosophical assumptions. It is, as he later says, a 'pre-philosophical' science. Although the justification of that claim would force us to define what *science* is, intuitively we can at least look at the *natural* sciences (not computer science, for example, which is as Brian Harvey says, neither about computers, nor a science) to help us decide where mathematics is like them. [*Questions.* Is mathematics a science like physics, biology, or chemistry? In what sense does it differ from them? In what sense is it like them? If mathematics is not a science in the sense of natural sciences, is it perhaps a science in the same way the two CSs (computer science, cognitive science) are sciences? If mathematics is indeed a science in one of those two senses (natural, CS-type), then is it reasonable to demand that mathematics should depend on as few philosophical assumptions as possible?]

The problem of mathematical truth. In the next two sections Curry discusses various answers to the question of what the *subject matter* of mathematics is. He mentions the three common views: realism, idealism, and formalism. In this section he discusses the first two, and then in the next section he describes his own (empirical) formalist view. The three views he describes as follows:

REALISM Mathematical propositions express the most general properties of our *physical environment*

Objections: (i) primitive, (ii) untenable (due to the role of ∞ in math)

IDEALISM Mathematics deals with the properties of *mental objects* of some sort

PLATONISM All infinistic constructions of classical mathematics are real

INTUITIONISM Depends on the existence of an *a priori* intuition of temporal succession

Objections: (i) vague, (ii) depend on metaphysical assumptions

FORMALISM Mathematics is the study of formal systems

[*Question.* Is Curry's presentation and criticisms of what he calls 'realism' and 'idealism' (in its two forms) reasonable? For example, can realism as he defined it be saved from the argument based on the necessity of infinistic reasoning in mathematics? Perhaps the Millians or Barkeleyians can give this defense a shot.] In the next two weeks we'll dig into the 'idealist' positions deeply, so we'll be in a better position to assess the purported (i) vagueness and the (ii) ontological extravagance of these positions. But any comments about these views or Curry's objections to them are welcome for discussion. As regards the description of formalism, he adds the following (204):

- ▷ The propositions of mathematics are the propositions of some formal system or set of systems
- ▷ The propositions of mathematics are either elementary or metatheoretic
- ▷ The propositions of mathematics, if not indefinite, have a *resolution process* (*Entscheidungsverfahren*[1])

We can discuss questions about these thesis of his position. In the next section he discusses the question of the *acceptability* of a formal system, which are basically the reasons that make the application of a formal system to a specific domain possible. He mentions and criticizes three such reasons defended by those who hold non (empirical) formalist views:

1. Self-evidence
2. Internal Consistency
3. Usefulness

Curry is a relativist/pluralist about acceptability: whether a formal system is acceptable is relative to a purpose. Even intuitionistic systems, he admits, can be acceptable for certain purposes.

[1]Related to the *decision problem*, but not exactly the same, of course.